Differential Geometry IV
Problem Set 10

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(1) Prove that every oriented Riemannian 4–manifold (X, g) admits a $\mathrm{spin}^{\mathrm{U}(1)}$–structure.

Hint: ***read*** `https://people.mpim-bonn.mpg.de/teichner/Math/ewExternalFiles/spin.pdf`

(2) Let (X, g) be an oriented Riemannian 4–manifold together with a $\mathrm{spin}^{\mathrm{U}(1)}$–structure $\mathfrak{w}$. For $(A, \Phi) \in \mathscr{A}(\mathfrak{w}) \times \Gamma(W^+)$ define the energy

$$E(A, \Phi) := \int_X |\nabla_A \Phi|^2 + \frac{1}{4}\mathrm{scal}_g |\Phi|^2 + \frac{1}{2}|\Phi|^4 + |F_A^{\mathrm{tw}}|^2.$$

Prove that Seiberg–Witten monopoles minimise E.

(3) Let (X, g) be an oriented Riemannian 4–manifold. Let $\eta \in \Omega^+(X, i\mathbf{R})$. Prove that there are only finitely many $\mathrm{spin}^{\mathrm{U}(1)}$–structures $\mathfrak{w}$ with $\mathscr{M}(\mathfrak{w}, \eta) \neq \varnothing$.

(4) Let H be an infinite-dimensional separable complex Hilbert space. Prove that $H^* := H \backslash \{0\}$ is contractible.