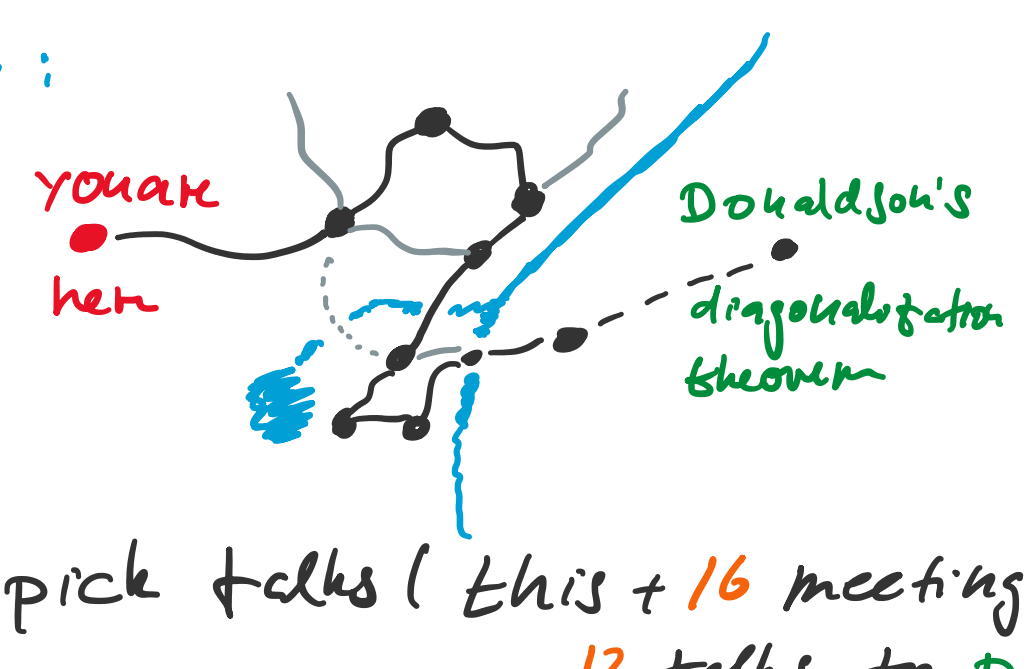
Gauge Theory Seminar: Overview and Organization

Sunday, November 1, 2020 10:10 AM

Plan for today:

* road map: (lost of talks)



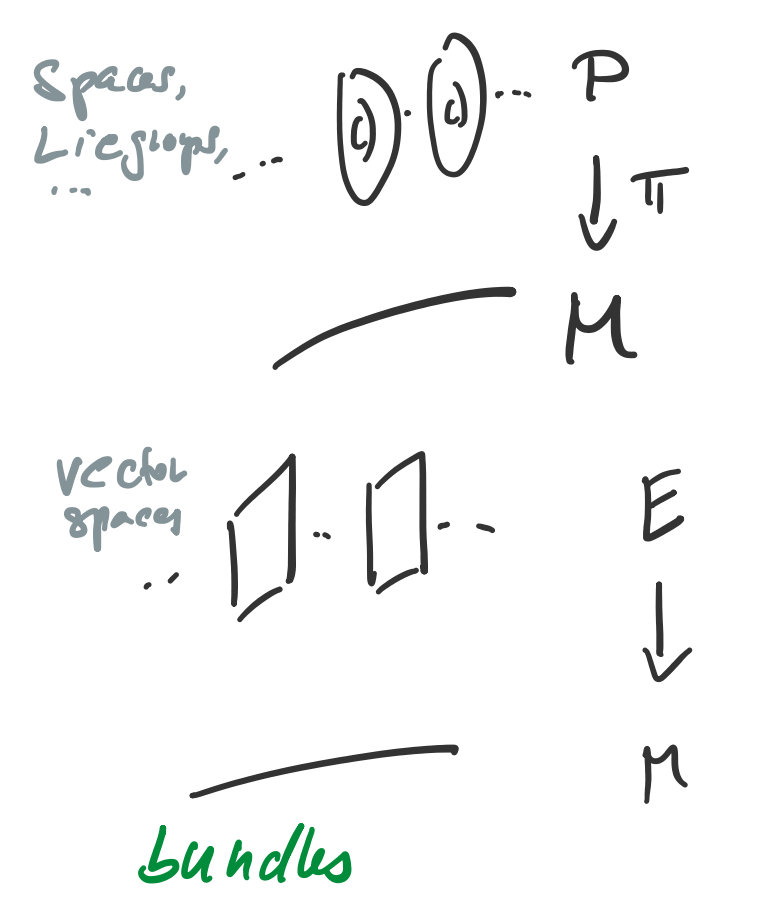
* participants pick talks (this + 16 meetings
  12 talks to DDT;
  5 further topics)

"Def":

(mathematical) gauge theory is the study of fiber bundles (vector, principal, ...), sections, connections; PDE involving these; moduli spaces of solutions; interactions with (low-dimensional) topology, physics, complex algebraic geometry, Riemannian geometry, symplectic geometry, representation theory, ...

I. geometric foundations

Talk 1: Bundles, connections, and curvature — Thomas



- gauge symmetry
- How to differentiate in this context?
  ⤳ connection, covariant derivative, parallel transport
- curvature

refs: Baum "HFB"; Freed-Uhlenbeck §2.1, Donaldson-Kronheimer §2.1

Talk 2: Characteristic classes, Chern-Weil theory

- classification of $P \xrightarrow{\pi} M$ in terms of homotopy theory $[M, BG]$
- $H^\bullet(BG)$ ⤳ characteristic classes of P $\gamma(P) \in H^\bullet(M)$
- construction of $\gamma(P)$ via connections Chern-Weil theory
- classification of $SU(2)$-bundles on 4-mfds

refs: Milnor-Stasheff; Freed-Uhlenbeck §2.2

Talk 3: Yang-Mills functional, ASD instantons, the BPST instanton — Niklas

- YM(H) natural energy functional in gauge theory
- connection w/ Maxwell's equation, monopoles, ...
- energy identities from CW-theory, instanton solutions
- explicit solutions (esp. BPST)

refs: BPST's paper, Freed-Uhlenbeck §2.3, §2.4

II. Analytic foundations

Talk 4: Sobolev spaces, Fredholm theory, and elliptic theory — Solveig

- develop a framework in which linear, elliptic differential operators (Δ, $d+d^*$, $\not{D}$) behave like finite dimensional matrices

[a speaker who already knows this and can present it packaged into usable black boxes would be ideal.]

[could be multiple talks if participants want details about inner workings]

Talk 5: Uhlenbeck's gauge fixing theorem — Douglas

- gauge symmetry ⤳ PDE are not elliptic
- break symmetry ⤳ gauge fixing
- Which gauge fixings give elliptic PDE?
- When can gauge fixing be achieved?

refs: Uhlenbeck's paper, Donaldson-Kronheimer §2.3, Freed-Uhlenbeck §8, Wehrheim's book

Talk 6: Uhlenbeck's removable singularity theorem — Shubar

- When can point-singularities in YM conn. be removed? ($4D$)

refs: Uhlenbeck's paper
better: proof via Otway's trick: cf. Smith-Uhlenbeck; I can explain it.

Talk 7: Compactness (or failure thereof) for ASD instantons — Naageswaran

- Understand compactness modulo bubbling.

refs: Freed-Uhlenbeck §8, Donaldson-Kronheimer §4.4, poss. Sedlacek's paper

Talk 8: Taubes' gluing theorem — Mircea

- Construct ASD instantons by gluing BPST instantons to flat backgrounds.

refs: Taubes' papers, Donaldson-Kronheimer §7.2, Freed-Uhlenbeck §6

III. The proof of Donaldson's diagonalization theorem

Mini-talk: Outline of the proof

Talk 9: Overview of topology of 4-mfds — Adrian

refs: Donaldson-Kronheimer §1

Talk 10: Construction of the moduli space of ASD instantons — (Joshua)

- Banach manifolds
- Fredholm maps
- Slices
- The Freed-Uhlenbeck theorem

refs: Donaldson-Kronheimer §4.1-§4.3, Freed-Uhlenbeck §3, §5

Talk 11: The collar theorem — Thomas

- Precise description of the boundary of the Uhlenbeck moduli space

refs: Freed-Uhlenbeck §9, Donaldson-Kronheimer §7.3

Talk 12: Conclusion of the proof/analysis of reducibles — Michael

refs: Donaldson's paper, Freed-Uhlenbeck §4, Donaldson-Kronheimer §8 ✓

IV. Further topics — 5 talks

* Taubes' construction of the Casson inv't (1-2 talks)
* gauge theory and cx. geometry, Donaldson-Uhlenbeck-Yau (2-4 talks)
* The ADHM construction of instantons on $\mathbb{R}^4$ (1-2 talks)
* Donaldson polynomials and more applications (1-∞ talks)
* higher dimensions (0-∞ talks) — Shubar