

# Differential Geometry III

## Gauge Theory

### Problem Set 5

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- (1) Consider the tangent group  $TG$ . Determine formulae for the group operation with respect to the two isomorphisms  $G \times \mathfrak{g} \rightarrow TG$  defined by

$$(g, \xi) \mapsto (g, TR_g(\text{ev}_1(\xi))) \quad \text{and} \quad (g, \xi) \mapsto (g, TL_g(\text{ev}_1(\xi))).$$

- (2) Prove that if  $(p: P \rightarrow B, R: P \times G \rightarrow P)$  is a  $G$ -principal fibre bundle, then  $(Tp: TP \rightarrow TB, TR: TP \times TG \rightarrow TP)$  is a  $TG$ -principal fibre bundle.
- (3) Let  $V \rightarrow B$  be a vector bundle of rank  $r$ . Suppose that  $\nabla$  is a covariant derivative on  $V \rightarrow B$ . Prove that there is a unique  $GL_r(\mathbf{R})$ -principal connection  $A$  on  $\text{Fr}(V) \rightarrow B$  such that  $d_A$  corresponds to  $\nabla$ .